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140. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Determine (any way) whether the Diophantine equation  $\left(\frac{2x-1}{3}\right) = x^2 + y^2$  has any positive integer solutions.

Solution by JACOB WESTLUND, Ph. D., Purdue University.

In order that  $\frac{2x-1}{3}$  shall be an integer we must have  $x=2+3a$ , where  $a$  is a positive integer. Hence  $(1+2a)^2 = (2+3a)^2 + y^2$  or after a few reductions  $y^2 = 8a^2 - 6a + 3(a^2 - 1)$ .

If  $a$  is odd, this equation is impossible, since in that case  $y$  must be even and hence all the terms except  $6a$  divisible by 4.

If  $a$  is even, we put the equation in the form  $y^2 = 8a^2 + 3a(a-2) - 3$ . This shows that  $y$  must be odd and  $y+3$  divisible by 8. Hence, setting  $y=2b+1$ ,  $4b^2 + 4b + 4$  should be divisible by 8 or  $b(b+1)+1$  divisible by 2, which is impossible. Hence the given equation has no positive integer solutions.

Also solved by A. H. Holmes.

No solution of 141 has yet been received.

#### AVERAGE AND PROBABILITY.

178. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Two random planes cut a given sphere. What is the chance that they intersect within the sphere?

I. Solution by HENRY HEATON, Belfield, N. D.

Let  $AB$  and  $CD$  be axes of the sphere perpendicular to the two planes and let  $EF$  be a trace of one of the planes.

Put  $x=OI$ , the distance of the plane through  $EF$  from the center of the sphere. Put  $\theta = \angle BOC$ . Then  $HG$ , the projection of  $EF$  upon  $CD=2\sqrt{(a^2-x^2)}\sin\theta$ .

It seems to be generally understood that the number of directions of the plane perpendicular to  $CD$  depends upon the number of different directions possible to  $CD$ , and that this depends upon the number of points in the surface of the sphere. Hence the number of planes of the direction  $\theta$  is proportional to  $\sin\theta$ . The angle  $\theta$  being supposed fixed the chance of intersection within the sphere is  $\frac{HG}{CD} = \frac{\sqrt{(a^2-x^2)}\sin\theta}{a}$ .

Hence the required probability is

$$p = \int_0^{\frac{1}{2}\pi} \int_0^a \sqrt{(a^2-x^2)} \sin^2\theta d\theta dx / \int_0^{\frac{1}{2}\pi} \int_0^a a \sin\theta d\theta dx$$

